Energetic-Ion-Driven MHD Instab. & Transport: Simulation Methods, V&V and Predictions

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Bulk plasma (fusion fuel)
MHD fluid model

Current $J$, Pressure $P$
Interactions
EM fields $E$, $B$

Energetic particles “EP”
(fusion $\alpha$, beams, heating)

Kineti c model

14.1 MeV

$\sim 10$ keV

3.5 MeV
Interplay of MHD and EP dynamics on multiple time scales

From structure formation to intermittent bursts

- **Long time scale**
  - 10-100 ms
  - Global profile build-up & collisional slow-down

- **Meso time scale**
  - 1-5 ms
  - Growth, chirping, decay

- **Short time scale**
  - 0.1-0.3 ms
  - Structure formation, beating

Intermittent instability bursts
Interplay of MHD and EP dynamics on multiple time scales

From structure formation to intermittent bursts

- **Long time scale**
  - Intermittent instability bursts
  - Growth, chirping, decay
  - Structure formation, beating

- **Meso time scale**
  - 0.1-0.3 ms
  - 1-5 ms

- **Short time scale**
  - 10-100 ms

Global profile build-up & collisional slow-down

(A) Steady state

(B) Relaxation events
1. Hybrid model (MEGA code)

MHD-kinetic hybrid model with collisions, sources & sinks for multi-time-scale simulations

Contents:
1. Hybrid model
2. Multi t scale: Bursting, chirping, beating
3. Short t scale: Stability and plasma response
4. Long t scale: Energetic particle confinement
5. Summary
MEGA code: Multi-time-scale hybrid model (0.01-100 ms)

**Bulk plasma: MHD model**
(t): 4th-order Runge Kutta, $\Delta t_{\text{mhd}} \approx 1$ ns
(R, $\phi$, Z): finite differences, non-slip b.c.

\[
\frac{\partial \rho_b}{\partial t} = -\nabla \cdot (\rho_b \delta \mathbf{u}_b), \quad \mu_0 J = \nabla \times \mathbf{B}
\]

\[
\rho_b \frac{\partial \mathbf{u}_b}{\partial t} = -\rho_b \mathbf{u}_b \cdot \nabla \mathbf{u}_b - \nabla p_b + (J - J_{\text{h,eff}}) \times \mathbf{B} - \left[ \nabla \times (\nu \rho_b \nabla \times \mathbf{u}_b) + \frac{4}{3} \nabla (\nu \rho_b \nabla \cdot \mathbf{u}_b) \right]
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad \mathbf{E} = -\mathbf{u}_b \times \mathbf{B} + \eta \delta \mathbf{J}
\]

\[
\frac{\partial p_b}{\partial t} = -\nabla \cdot (p_b \mathbf{u}_b) - (\Gamma - 1) \left[ p_b \nabla \cdot \mathbf{u}_b + \eta (J - J_{\text{h,eff}}) \delta \mathbf{J} \right] + \nu \rho_b (\Gamma - 1) \left[ \nabla \times \mathbf{u}_b \right]^2 + \frac{4}{3} \left[ \nabla \cdot \mathbf{u}_b \right]^2 + \chi \nabla^2 p_b
\]

**Energetic ions: Kinetic model**
(t): 4th-order Runge Kutta, $\Delta t_{\text{pic}} \geq \Delta t_{\text{mhd}}$

(f$_{\text{gc}}$): PIC, $\delta f$ or full-f guiding center distribution

\[
\frac{d R_{\text{gc}}}{dt} = -\frac{\mu}{q B^*} \nabla B \times \mathbf{b} + \frac{v_{||}^*}{B^*} (B + \rho_{||} B \nabla \times \mathbf{b}) + \frac{E \times \mathbf{b}}{B^*} \equiv U_{\text{gc}}
\]

\[
m v_{||} \frac{d v_{||}}{dt} = v_{||}^* (q E - \mu \nabla B)
\]

\[
d \frac{d \mathbf{u}}{dt} = 0 + O(\beta \epsilon_b) \quad \text{with} \quad \epsilon_b \sim \frac{\Omega_{L}}{L_B} \sim \Omega_L \ll 1
\]

\[
\mu = \frac{m v_{L}^2}{2 B}, \quad \rho_{||} = \frac{v_{||}}{\omega_L}, \quad B^* = B \left[ 1 + \rho_{||} \hat{b} : (\nabla \times \hat{b}) \right], \quad \hat{b} = \frac{\mathbf{B}}{B}
\]

\[
v_{||}^* = \frac{v_{||}^*}{v} v + \Delta v_{L} + \frac{v_{||}}{v} \Delta v_{T} \sin \Omega, \quad v_{\perp} = \sqrt{v_{L}^2 + \Delta v_{L}^2 + \Delta v_{T}^2 - v_{||}^2}
\]

**Shear Alfvén, slow and fast waves, resistive, viscous, thermal diffusion.**

**Wave-particle interactions**

**Guiding center \parallel streaming, \perp drifts, gyroaverage, collisions, sources, wall.**

**Energetic particle (EP) motion**
2. Multi time scale:

Short and long time scale dynamics self-consistently linked on meso time scale

Contents:
1. Hybrid model
2. Multi time scale
3. Short time scale
4. Long time scale
5. Summary
Simulation of multiple bursts of chirping modes in JT-60U

Self-consistent simulation including MHD, realistic EP source and collisions.

Reproduced: Experimentally observed EP-driven Alfvén modes with burst periods of 5-10 ms, chirping on 1 ms scale, and global beating on 0.1-0.3 ms scale.

→ Successful validation (qualitatively, quantitatively, on multi-t scales)
→ Enables to clarify underlying physics numerically ... but expensive:

- Took 40 days on 4096 cores (Helios) to simulate 35 ms with $\Delta t=1$ ns time steps.
3. Short time scale:

Stability and plasma response

Contents:
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Transition from weak to strong EP transport in JT-60U

Peak amplitude of EP-driven n=3 shear Alfven waves as function of drive strength:

[Bierwage et al, NF'14, NF'16]

EP pressure field in R-Z plane:

75 μs later

Demonstrated:
Possibility of distinct transitions from weak to strong EP transport ...
... even for a long-wavelength mode with single toroidal harmonic (n=3).

Potential relevance:
This may play a role in triggering relaxation events (EP avalanches).

Normalized EP pressure

Time
Predict Alfvén mode excitation and EP transport in ITER

3D global simulation of ITER-scale plasmas are highly demanding computationally.

Only short time window simulated with MEGA.

Prediction: Resonant modes cause benign or negligible EP transport

1. Most unstable modes:
   Short wavelength (n=10-20)
   high frequency (TAE)

2. Dominant mode in advanced nonlinear phase:
   Longer wavelength (n=3)
   lower frequency (BAE)

ITER 9 MA scenario

→ Encouraging result for ITER ...

... but remains to be verified, because initial EP profiles were not computed self-consistently.
4. Long time scale:

Confinement of energetic particle subject to...

(A) steady fluctuations,
(B) large relaxation events

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"Multi-phase" method for long-time simulations [Todo NF’14]

Interlaced classical (no MHD) and hybrid (with MHD) simulation phases.

Speed-up by \( \times (2 \cdots 5) \) gives access to long time scale of steady-state formation (> 50 ms).

Example: JT-60U with 5 MW of N-NBI at 400 keV

Benchmark study submitted to CPC. → See poster.
Application of multi-phase method

(A) Steady moderate Alfvénic fluctuations in DIII-D tokamak
(General Atomics, San Diego, USA)

[Todo et al, NF'15]
(A) Steady moderate Alfvénic fluctuations in DIII-D tokamak

Comparison between self-consistently simulated $\delta T_e$ fluctuations and ECE measurements in beam-driven DIII-D tokamak plasma:

[Todo et al, NF'15]

**Found:** For mode with longest wavelength ($n=3$) frequency and mode structure agrees well with experiment.

$\delta T_e (R,Z)$

Amplitude profile

Global structure

Phase profile

$\delta T_e [\text{eV}]$

Simulation $f=69\text{kHz}$

Experiment $f=74\text{kHz}$

$\text{Phase [rad] / } \pi$

Simulation $f=69\text{kHz}$

Experiment $f=74\text{kHz}$

$0.2$ $0.3$ $0.4$ $0.5$ $0.6$ $0.7$ $0.8$ $0.9$ $1$

$0$ $0.5$ $1$

$-1$ $-0.5$ $0$

Successful validation in regime where use of MHD is justified.
(A) Improved prediction for EP pressure profile in DIII-D

Comparison of steady-state EP pressure profiles: [Todo et al, NF'15]
- $P_{\text{exp}}(r)$ estimated from experimental measurements,
- $P_{\text{class}}(r)$ from “classical” Monte-Carlo simulation (EP source + collisions)
- $P_{\text{multi}}(r)$ from multi-phase simulation (interlaced hybrid and classical)

Found: Steady low-amplitude Alfvén mode activity reduces EP pressure in the plasma core by up to 60%.
Multi-phase sim. result is very close to exp. error bars.

→ This result together with successful validation of $\delta T_e$ amplitudes convincingly confirms that transport is caused by Alfvén modes.
Application of multi-phase method

(B) Abrupt Large relaxation Events (ALE) in JT-60U tokamak (JAEA, Naka, Japan)

[Bierwage et al, IAEA FEC 2016]

\[t=0: \text{Start of beam injection}\]

\[\beta_{EP} \text{ vs. Time}\]
SUMMARY

Within the regime where MHD is valid (long wavelength, high frequency) the hybrid code MEGA has largely succeeded in the simulation of MHD and EP dynamics, and their interplay on a wide range of t-scales.

0.1-0.3 ms

1-5 ms

Global profile build-up & collisional slow-down

10-100 ms

(A) Steady state

(B) Relaxation events

0.1-0.3 ms
OUTLOOK: ☐ Reproducibility ☐ Application ☐ Extension

Within the regime where MHD is valid (long wavelength, high frequency) the hybrid code MEGA has largely succeeded in the simulation of MHD and EP dynamics, and their interplay on a wide range of t-scales.

Try to reproduce with modified plasma and simulation parameters to check reliability.

→ Work in progress

Extend to low-frequency regime where shear Alfvén waves couple strongly with ion sound waves.

→ Kinetic bulk ion model
- Straightforward extension for kin. compression.
- Difficult to capture FLR effects (i.e., ITG, KBM, drift-Alfven micro-turbulence)

Apply to study & explain observations (e.g. ALE trigger mechanism).

Use insights to develop efficient reduced models for predictive simulations.
Appendix
Performance of “multi-phase” method

[Bierwage & Todo, submitted to CPC]
Benchmark of “multi-phase” method

- Agreement:
  n=1 dominant, followed by n=2

- Disagreement:
  Multi-phase sim. amplitudes overshoot, as they must in order to cause similar transport in 1/5 of the time.

[Bierwage & Todo, submitted to CPC]
Benchmark of “multi-phase” method

EP beta and stored energy

- Hybrid: 3.5%
- Multi-phase: 4.5%

Negligible difference w.r.t. confined EPs but significant for wall heat load.

EP beta profile and velocity distrib.

- EP distributions agree well
- Note difference between $\beta_{EP}(R)$ [local] and $\beta_{EP}(r)$ [flux-surf. avg.]