Energetic particle modes of $q=1$ high order harmonics in tokamak plasmas with monotonic weak magnetic shear

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• Background

• M3D-K model and basic parameters

• Linear results

• Nonlinear results

• Summary
Background

- Energetic particle physics is a key issue in burning plasma experiments of advanced tokamaks, such as the International Thermonuclear Experimental Reactor (ITER).

- The weak magnetic shear is recommended for high-performance tokamak operations, since the flat q-profile in the core region can substantially eliminate the free energy of the current gradient, consequently prevent sawtooth crashes. However, some instabilities, such as long lived modes, ballooning and infernal, can be excited in the weak magnetic shear configuration.

- The phenomenon that the high order harmonics stronger than the n=1 component in the low magnetic shear have been observed in some devices, such as HL-2A.
Wang et al. numerically investigated the q-profile effect on high-order harmonics q=1 tearing mode during the sawtooth crash\textsuperscript{1,2}. It is found that the q=1 high order harmonics can play crucial roles during the sawtooth oscillations and trigger the neoclassical tearing mode, which may lead to the disruption by supplying the seed island.

However, the theoretical and numerical investigation of the q=1 high-order energetic particle modes has made little progress.

\textsuperscript{1}Z. X. Wang, L. Wei, and X. G. Wang, Phys. Plasmas \textbf{19}, 062108 (2012).

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Physical model: M3D-K

Resistive MHD model:

\[
\rho \frac{d\mathbf{v}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p + \mu \nabla^2 \mathbf{v}
\]

\[
\frac{dp}{dt} = -\gamma p \nabla \cdot \mathbf{v} + \rho \nabla \cdot \left( \kappa \frac{\nabla p}{\rho} \right)
\]

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}
\]

\[
\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}
\]

\[
\mu_0 \mathbf{J} = \nabla \times \mathbf{B}
\]

\[
\nabla \cdot \mathbf{B} = 0
\]

kinetic/MHD hybrid model:

\[
\rho \frac{d\mathbf{v}}{dt} = -\nabla P - \nabla \cdot \mathbf{P_h} + \mathbf{J} \times \mathbf{B}
\]

\[
\mathbf{P_h} = P_\perp \mathbf{I} + (P_\| - P_\perp) \mathbf{b} \mathbf{b}
\]

\[
P_\|(\mathbf{x}) = \int m v_\|^2 \delta(\mathbf{x} - \mathbf{X} - \rho_h) F(\mathbf{X}, v_\|, \mu) B^{**} d^3 X d\mu d\theta
\]

\[
P_\perp(\mathbf{x}) = \int \frac{1}{2} m v_\perp^2 \delta(\mathbf{x} - \mathbf{X} - \rho_h) F(\mathbf{X}, v_\|, \mu) B^{**} d^3 X d\mu d\theta
\]

Gyrokinetic/drift-kinetic Equations:

\[
\frac{d\mathbf{X}}{dt} = \frac{1}{B^{**}} \left[ v_\| \mathbf{B}^* - \mathbf{b}_0 \times (\langle \mathbf{E} \rangle - \frac{1}{e} \mu \nabla (B_0 + \langle \delta \mathbf{B} \rangle)) \right]
\]

\[
m \frac{dv_\|}{dt} = \frac{e}{B^{**}} \mathbf{B}^* \cdot (\langle \mathbf{E} \rangle - \frac{1}{e} \mu \nabla (B_0 + \langle \delta \mathbf{B} \rangle))
\]

\[
\mathbf{B}^* = B_0 + \langle \delta \mathbf{B} \rangle + \frac{mv_\|}{e} \nabla \times \mathbf{b}_0
\]

\[
B^{**} = \mathbf{B}^* \cdot \mathbf{b}_0
\]
Basic parameters and initial profiles

Main parameters:

aspect ratio : $R_0/a=3.6667$.

elongation: $K=1.0$

triangularity: $\delta=0.0$

central total beta: $\beta_{total,0} = 0.8\%$

central beam ion beta: $\beta_{hot} / \beta_{total} = 0.5$

Toroidal magnetic field: $B_0=1.37T$

Beam ion distribution function:

$$f = \frac{cH(v_0-v)}{v^3 + v_c^3} \exp(-(\Lambda - \Lambda_0)^2 / \Delta \Lambda^2) \exp(-\langle \Psi \rangle / \Delta \Psi)$$

$$\Lambda \equiv \mu B_0 / E \quad \Lambda=1.0, \quad \Delta \Lambda=0.1, \quad \Delta \Psi=0.3$$

The normalized beam particle speed and gyroradius are given by:

$$\nu_h / \nu_A = 0.3966, \quad \rho_h / a = 0.0645$$

Equilibrium profiles of pressure and safety factor. And the plasma density profile is uniform in space.
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Linear results

- It is found that the mode is stable in the MHD limit. Furthermore, it is observed that when the kinetic effects of beam ions are included, the high-order harmonics have a larger growth rate than the m/n=1/1 harmonic.

Linear mode structure (stream function U)
The solid line shows the q=1 surface location.
Linear results: resonant condition analysis

- For the high-order harmonics, they satisfy multiple resonant conditions, while the n=1 component has only one resonant condition, which are quite different from the typical fishbone instability.
Linear results

Excitation conditions of high-order harmonics

• For a flat q-profile in the core region, the high order harmonics modes can be excited.

• Each mode is excited when the fast ion pressure exceeds a threshold, which indicates that energetic trapped particles play an essential role for the destabilization.

• The high-order harmonics stronger than the $m/n=1/1$ mode can be excited in low magnetic shear with a higher beam ion pressure.
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Nonlinear results: fluid nonlinearity effect

- The fluid nonlinearity reduces the saturation level of the m/n=1/1 component, while it hardly affects high-n components, especially the modes with m=n=3,4.
- The maximum values of kinetic energy of high-order harmonics are all larger than m/n=1/1, while the m/n=1/1 mode is dominant in the saturated phase.
Nonlinear results: mode frequency evolution

- **n=1**: the frequency chirps down continuously.
- **n=2**: the frequency firstly stays almost unchanged, and then shifts upward about 10% and later keeps constant again afterwards.
- **n=3** and **n=4**: the frequencies firstly keep almost constant values, and then shift downward about 10%, and then keep constant again with time.
Nonlinear results: 1D distribution of fast ions

\begin{align*}
\text{(a) } n=1 & \quad \mu \approx 0.8 \text{ and } E = 0.76; \\
\text{(b) } n=2 & \quad \mu \approx 1.5 \text{ and } E = 1.43.
\end{align*}

- The flattening region of energetic particle distribution function due to separately high order harmonics excitation is wider than that due to \(m/n=1/1\) component.
Nonlinear results: 2D distribution of fast ions

- In comparison with the $n=1$ component, there is a larger flattening region of distribution function for the high-order harmonics.

- When fluid nonlinearity is included, the flattening region is larger than that without fluid nonlinearity.
Summary and Perspectives

- **Linear results:**
  - For a flat q profile in the core region, the linear growth rate of high order harmonics driven by energetic trapped particles can be higher than the $n=1$ component.
  - There exist multiple resonant locations satisfying different resonant conditions in the phase space of energetic particles for the high $n$ components.

- **Nonlinear results:**
  - The fluid nonlinearity reduces the saturation level of the $n=1$ component, while it hardly affects high $n$ components.
  - The flattening region of energetic particle distribution due to high-order harmonics excitation is wider than that due to $n=1$ component, although the $n=1$ component has higher saturation amplitude.

- **The mechanism of high order harmonic modes driven by passing energetic particles in the weak magnetic shear configuration will be considered in future.**
Thank you for your attention!